Buckling and Postbuckling of Orthotropic Plates

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Theme

THE stability is considered of flat rectangular plates when in-plane biaxial loads are applied. The in-plane and bending elastic properties are orthotropic and the plate axes of symmetry and the elastic axes of symmetry coincide. Both the initial buckling and the reduction in stiffness occurring at the instant of buckling are considered. Numerous authors have considered the stability of orthotropic plates under biaxial load. Wittrick,[1] for example, has shown for the simply supported case among others that the parameters involved can be combined into one curve giving the buckling load for any plate. Reference 2 considers the buckling and postbuckling of a class of biaxially loaded laminated plates in the presence of bending-extensional elastic couplings. The present paper treats the stability of specially orthotropic plates with these couplings absent; a further rationalization is then possible, in which for any plate the proportional loss of stiffness at buckling can also be expressed in a single curve. For a particular plate, the combination of these universal curves, for buckling load and the loss of stiffness at buckling, allows unified consideration of the stability of simply supported biaxially loaded orthotropic plates. It follows that the relative stiffness at buckling (i.e., the ratio of plate stiffness immediately after buckling to that immediately before) depends on one parameter involving the elastic constants and the transverse load; this fact may be exploited in design.

Contents

The governing equations are those that control the large-deflexion response of a flat specially orthotropic plate, $0 \leq x \leq a$, $0 \leq y \leq b$, thickness h, under loading $(N_x N_y)$. Only the in-plane stiffnesses A_{11}, A_{12}, A_{22}, A_{33} and the bending stiffnesses D_{11}, D_{12}, D_{22}, D_{33} are present, all others vanishing; the underlying theory is given elsewhere.[2]

Initial buckling occurs when

$$-b^2\pi^{-2}[D_{11}D_{22}(1-k)]^{-1/2}(N_x-N_x') = (X-X^{-1})^2 \quad (1)$$

where

$$k=-b^2N_y/\pi^2D_{22} \quad (2)$$

is the fraction (applied transverse load)/(wide plate buckling load),

$$N_x' = -2\pi^2b^{-2}\{D_{12}+2D_{33}+[D_{11}D_{22}(1-k)]^{1/2}\} \quad (3)$$

is the buckling load of a preloaded long plate, and

$$X=a[D_{22}(1-k)]^{1/4}/mbD_{11}^{1/4} \quad (4)$$

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Equation (1) provides a particularly simple relationship between the buckling load and the parameter X that holds for all orthotropic plates of present type and also holds for isotropic plates.[1]

When using Eq. (1) for the variation of buckling load with a/b, the number of axial half-waves m must be accounted for Figure 1 shows the typical variation of N_x/N_x' with $a[D_{22}(1-k)]^{1/4}/bD_{11}^{1/4}$. Being derived from Eq. (1), successive branches $m=1,2,3,\ldots$are identical in general shape, only differing by lateral scaling factors proportional to m. At the point A_m in Fig. 1 where the m-branch and the $(m+1)$-branch intersect

$$N_x=-\pi^2b^{-2}\{2(D_{12}+2D_{33})+[m(m+1)^{-1} + m^{-1}(m+1)]\times[D_{11}D_{22}(1-k)]^{1/2}\} \quad (5)$$

$$a[D_{22}(1-k)]^{1/4}/bD_{11}^{1/4}=[m(m+1)]^{1/2} \quad (6)$$

The minimum buckling load on each branch, labeled B, occurs when

$$a[D_{22}(1-k)]^{1/4}/bD_{11}^{1/4}=m \quad (7)$$

with $N_x=N_x'$. The transitions A_m thus always occur at values $2^{1/2}$, $6^{1/2}$, $12^{1/2}$...and the mimimum buckling loads at values $1,2,3,\ldots$of the parameter mX.

The large-deflexion plate equations also provide the loss of axial stiffness at buckling[2]; the principles are based on Koiter's initial postbuckling theory,[3] and the fact that the initial mode predominates following buckling.

If the boundary displacements are uniform during loading, the relative stiffness (the ratio of plate stiffness immediately after buckling to that immediately before) is[2]

$$\text{relative stiffness} = 1-2(3+Y^4)^{-1} \quad (8)$$

$$\text{where} \qquad Y=aA_{22}^{1/4}/mbA_{11}^{1/4} \quad (9)$$

Equation (8) constitutes a further simple functional relationship, between the relative stiffness and the parameter Y, which is true for all plates of the present type; since it is a monotonically increasing function, higher stiffnesses are always associated with greater values of Y.

Equations (1) and (8) are similar in kind, relating buckling properties for all plates to single, but different, non-dimensional wavelength parameters; X and Y are themselves related by a scaling

$$\mu=\{A_{22}D_{11}/A_{11}D_{22}(1-k)\}^{1/4} \quad (10)$$

which involves the plate properties via A_{11}, A_{22}, D_{11} D_{22}. Once μ is known, a unique relationship exists between points on the buckling and the stiffness curves. Thus, the minimum buckling load (B in Fig. 1) always corresponds to

$$\text{relative stiffness} = 1-2(3+\mu^4)^{-1} \quad (11)$$

depending only on μ.

Figure 1 shows, for a range of μ, the variation of relative stiffness over the different segments of the buckling curve;

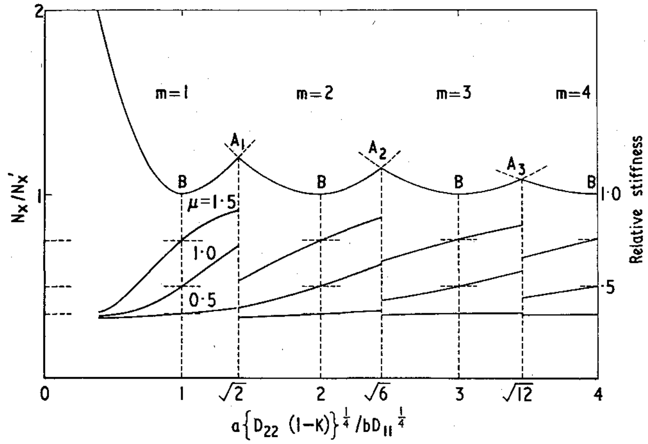

Fig. 1 Variation of buckling properties with plate geometry.

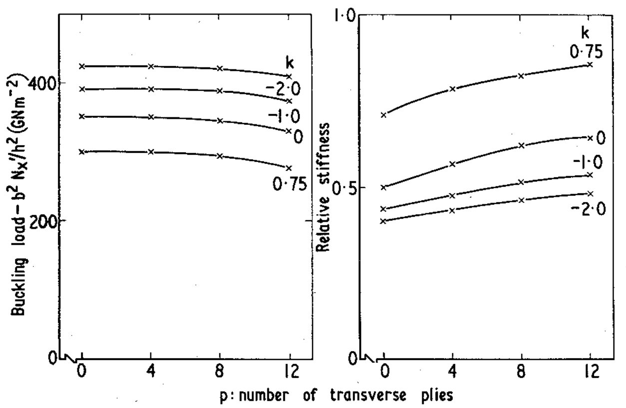

Fig. 2 Buckling load and relative stiffness of a balanced 24-ply long plate.

these stiffness curves will be universal for all plate properties and preloads associated with that μ. At the mode transition points A_m a fall in stiffness occurs on passing from the m-branch to the $(m+1)$-branch; this fall is from a value of $1-2[3+m^{-2}(m+1)^2\mu^4]^{-1}$ in mode m to one of $1-2[3+m^2(m+1)^{-2}\mu^4]^{-1}$ in mode $(m+1)$. These expressions again depend only on μ.

A case of interest occurs if $A_{22}D_{11}=A_{11}D_{22}$, when

$$\mu=(1-k)^{-1/4} \tag{12}$$

Since (A_{ij}) and (D_{ij}) are proportional in a homogeneous plate, this corresponds to the homogeneous orthotropic or isotropic plate. The curve $\mu=1$ (Fig. 1) can thus be regarded as corresponding to a homogeneous plate in the absence of transverse load; it will also correspond to heterogeneous plates when $k\neq 0$.

In laminated plates, the relative stiffness is a design parameter that is not available in homogeneous plates; by the above analysis its value increases with μ. A figure of merit is provided by the homogeneous plate stiffness since departures from this value may occur because of ply layup. For the long homogeneous plate,

$$\text{relative stiffness} = (2-k)(4-3k)^{-1} \tag{13}$$

from Eqs. (10) and (11). The long-plate stiffness, Eq. (11), will exceed this value when $D_{11}A_{22}/A_{11}D_{22}>1$.

The way in which different ply orientations contribute to plate stiffnesses is well known. Axial plies contribute mainly to (A_{11}, D_{11}), transverse plies to (A_{22}, D_{22}), whereas $\pm 45°$ plies contribute mainly to $(A_{12}, A_{33}, D_{12}, D_{33})$. $D_{11}A_{22}/A_{11}D_{22}$ may be increased either by increasing D_{11}/A_{11} (e.g., by placing axial plies near the plate upper and lower surfaces for bending efficiency) or by decreasing D_{22}/A_{22} (e.g., by placing transverse plies near the middle surface). Resistance to buckling will be provided by $\pm 45°$ plies disposed efficiently in bending; longitudinal plies react the axial load, and enhance the relative stiffness if near the plate surfaces.

The illustrative example uses a balanced 24-ply laminate with fiber directions ($\pm 45°$, $90°$). Although such laminates are not generally specially orthotropic, this becomes of lesser significance with a large number of plies present. For the example, the elastic constants D_{13}, D_{23} are constrained to vanish; a separate check shows this to give insignificant differences in buckling load and wavelength.

The variations of buckling load N_x', Eq. (3), and relative stiffness, Eq. (11), are shown in Fig. 2 for a ($\pm 45°$, $-45°$,..., $-45°$, $90°$..., $90°$/symmetric) 24-ply laminate having p transverse plies at the center of the plate. The ply properties used are $E_1=207$ GNm^{-2}, $E_2=7.58$ GNm^{-2}, $G=5.52$ GNm^{-2}, $\upsilon_{12}=0.3$. When $p=0$, this layup is pseudoisotropic in-plane with a high buckling efficiency. Since central plies contribute little in bending, only a slight fall-off in buckling load occurs with increasing p; when $k=0$, for example, a reduction over $p=0$ occurs of 2% ($p=8$) and of 6% ($p=12$). These small reductions in buckling load are associated with relative stiffness increases of 24% ($p=8$) and of 30% ($p=12$); since $p=0$ gives a nonhomogeneous plate having $A_{11}D_{22}=A_{22}D_{11}$, these relative stiffness values correspond to the yardstick of the homogeneous plate by which improvements in stiffness may be judged. The transverse load affects this improvement, which for high values of k will be less since the homogeneous plate stiffness is already high (see Eq. 13). For stabilizing transverse loads (e.g., $k=2$, Fig. 2) lesser reductions in buckling loads combine with lesser improvements in the relative stiffness.

References

[1]Wittrick, W. H., "Correlation Between Some Stability Problems for Orthotropic and Isotropic Plates under Biaxial and Uniaxial Direct Stress," *Aeronautical Quarterly*, Vol. 4, 1952, pp. 83–92.

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[3]Koiter, W. T., "On the Stability of Elastic Equilibrium," Thesis, Delft, 1945, English translation, NASA-TT-F-10833, 1967.